Lecture 2 Summary wed 30/07/14

Vocabulary

* sequence that converges to x
* metric space

Examples

METRIC SPACES

(1) $X = \mathbb{R}^2 = \{(z,w) \mid z \in \mathbb{R}, w \in \mathbb{R}\}$ with $d : X \times X \to \mathbb{R}_{\geq 0}$ given:

$$\begin{aligned} d(x,y) &= |y-x| \\ &= |(y_1,y_2)-(x_1-x_2)| \\ &= |(y_1-x_1, y_2-x_2)| \\ &= \sqrt{(y_1-x_1)^2+(y_2-x_2)^2}, \text{ where } x=(x_1,x_2) \text{ and } y=(y_1,y_2) \end{aligned}$$

(2) $$\begin{aligned} \mathbb{R}^n &= \{(x_1, x_2, \dots, x_n) \mid x_i \in \mathbb{R}\} \\ &= \{x : [1,n]_{\mathbb{Z}} \to \mathbb{R}\} \\ &\qquad\quad i \mapsto x_i \end{aligned}$$

NORMS

- $\|x\| = \sqrt{x_1^2+x_2^2+\dots+x_n^2}$
  $= (x_1^2+x_2^2+\dots+x_n^2)^{\frac{1}{2}}$ if $x=(x_1,x_2,\dots,x_n)$

- $\|x\|_p = (|x_1|^p+|x_2|^p+\dots+|x_n|^p)^{\frac{1}{p}}$

- $\|x\|_\infty = \sup\{|x_1|, |x_2|, \dots, |x_n|\}$

Homework

- Changing $d : X \times X \to \mathbb{R}_{\geq 0}$ and axioms? ie. why is the definition of metric space the "right" one?

- Definition of convergence? $x \in X$?

- In $\mathbb{R}^\infty$, what if $\|x\|_p = \left(\sum_{i=1}^{\infty}|x_i|^p\right)^{\frac{1}{p}}$ doesn't exist?